System Stability

CEG3H3

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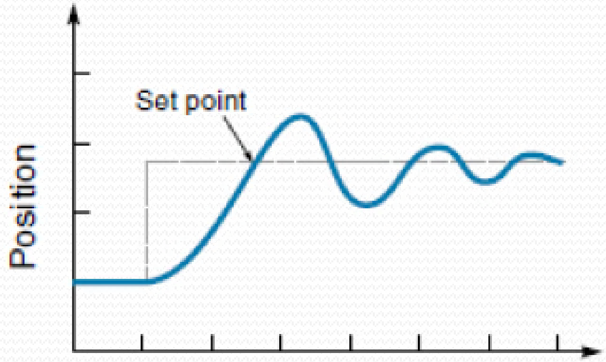
Objectives

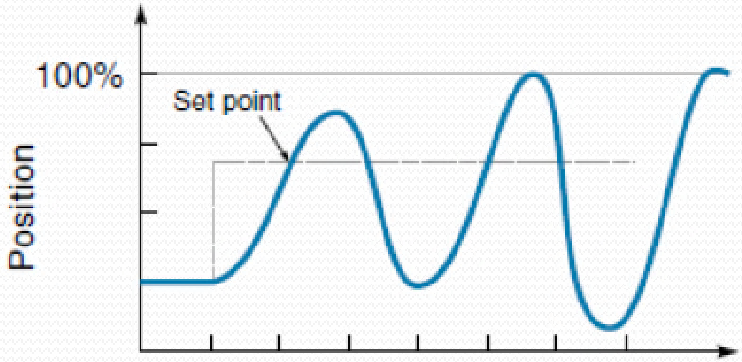
- Understand the concept of stability and interpret a Bode plot, then predict its gain and phase margin
- Implement two methods of tuning a process control system and calculate the PID constants

Stability

- A stable system is one where the controlled variable will always settle out at or near the set point
- An unstable system is one where, under some conditions, the controlled variable drifts away from the set point or breaks into oscillations that get larger and larger until the system saturates on each side



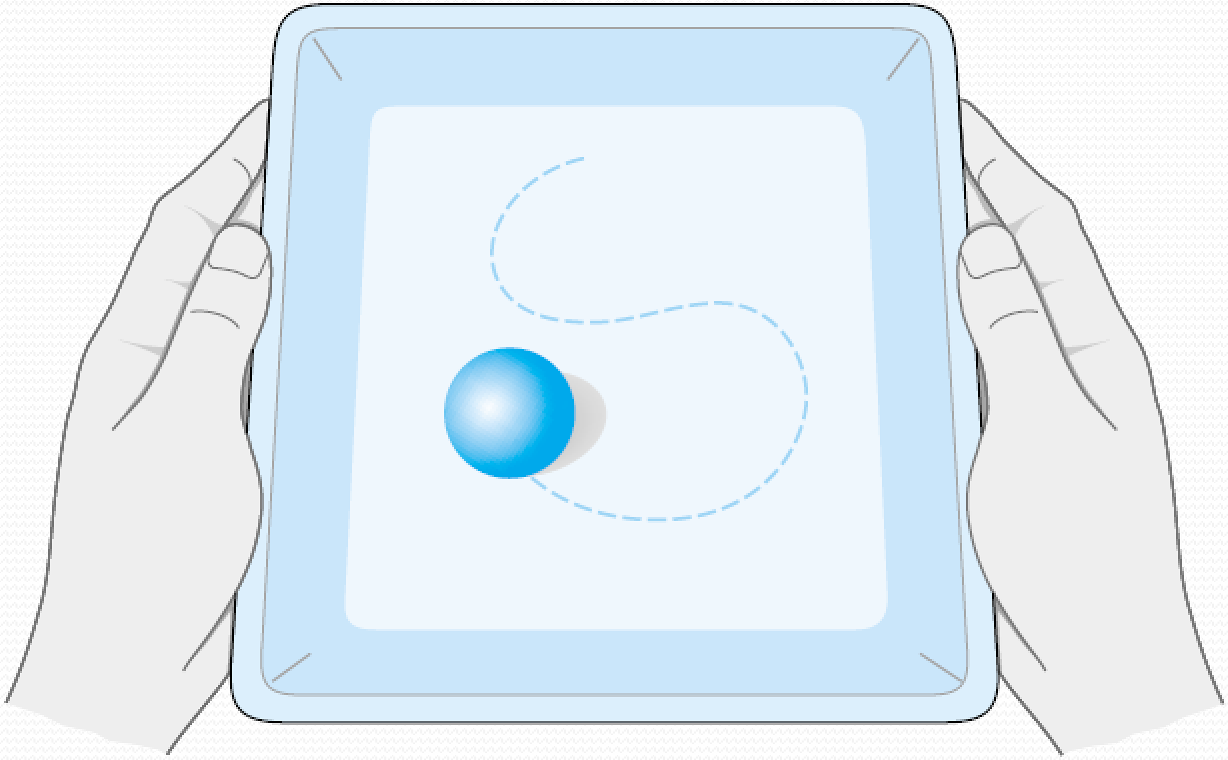
(a) Stable system



(b) Unstable system

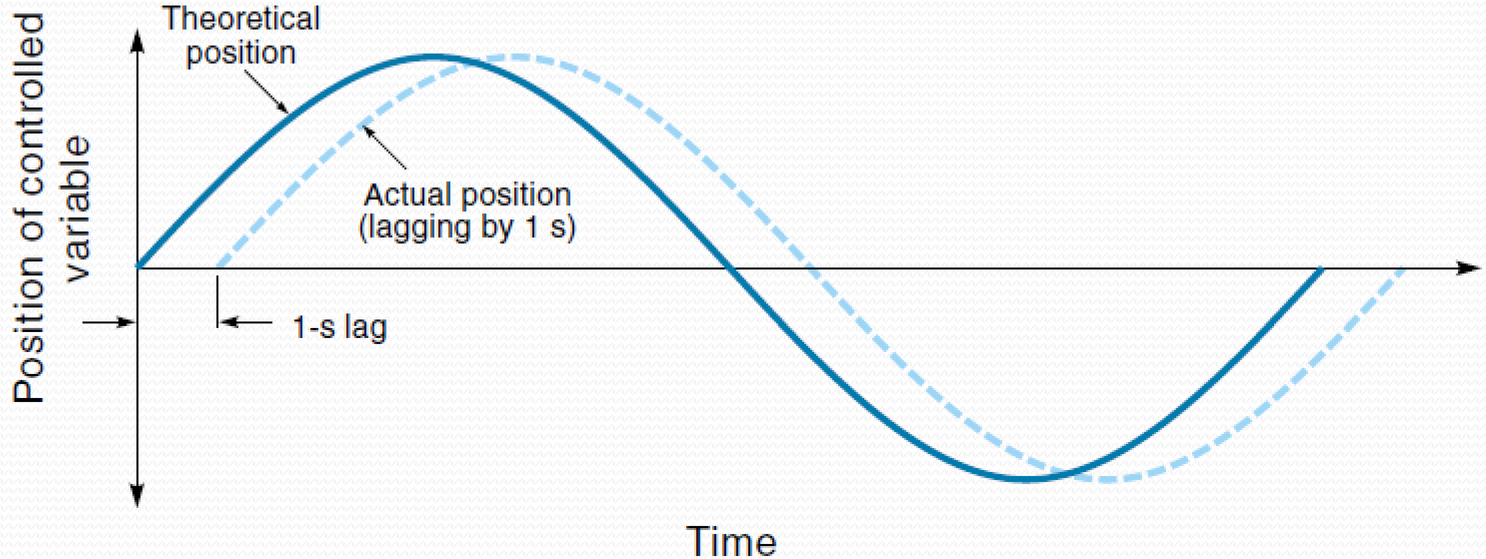
Oscillation Problem

- Caused by lagging, dead time, or backlash



Oscillation Problem

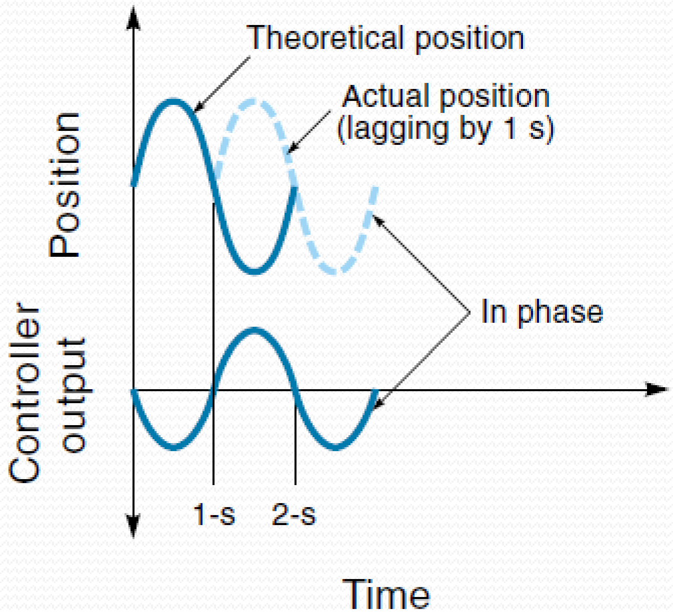
- Lag time causes phase lag, but the amount of phase lag depends on the frequency



(a) System with long cycle time; 1-s lag is not significant.

Oscillation Problem

- Lag time causes phase lag, but the amount of phase lag depends on the frequency



(b) System with 2-s cycle time; 1-s lag causes 180∞ lag.

Bode Plot

- Bode plot is a graph that can help determine whether a system is stable or not
- To make a stable system, the gain must be less than 1 for any oscillating frequency where the lag causes an additional 180° of phase shift

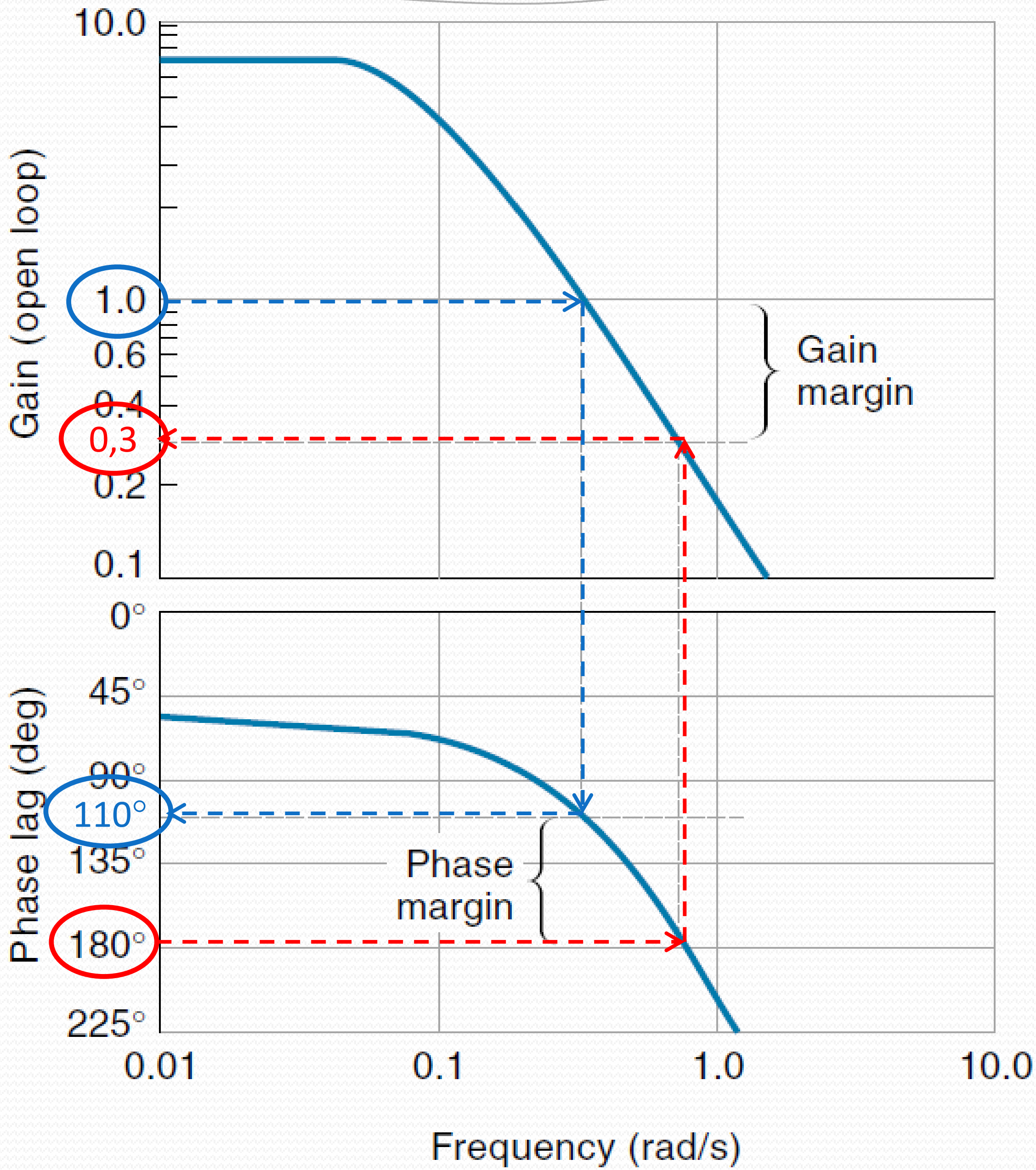
Bode Plot

- Gain margin :

  1,0 – 0,3 = 0,7

- Phase margin :

  180° – 110° = 70°

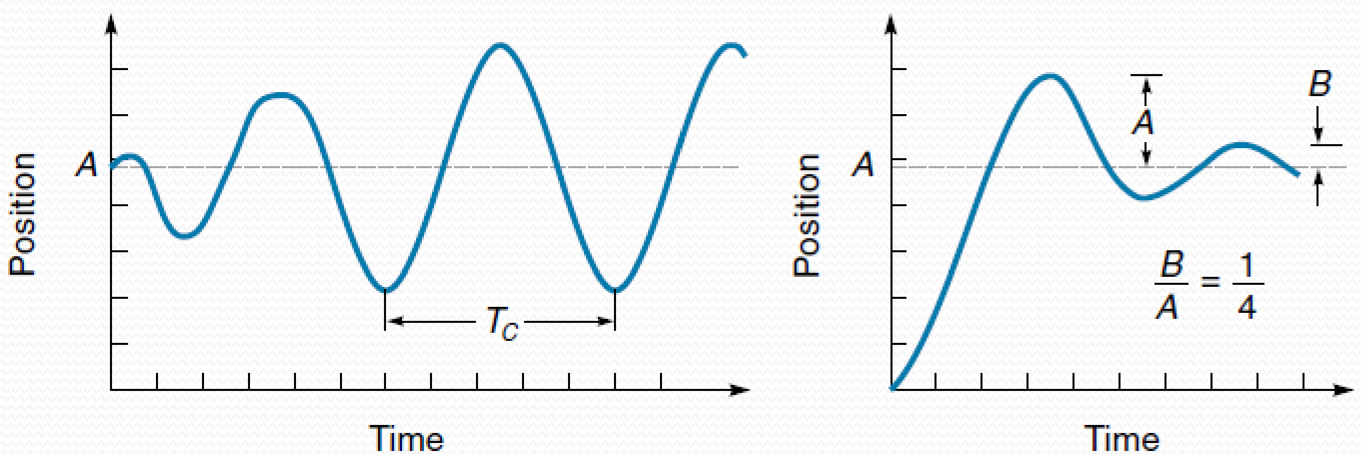


Tuning PID Controller

- Method of arriving at numerical values for the constants K_P, K_I, and K_D
- Many methods exist, but two of the most common were developed by Zieler and Nichols :
  - Continuous-cycle method
  - Reaction-curve method

Continuous-cycle method

- Can be used when harm isn't done if the system goes into oscillation
- Will yield a system with a quick response (means a step-function input will cause a slight overshoot that settles out very quickly)



(a) System as forced into oscillation (b) Resulting response after tuning

Continuous-cycle method

1. Set $K_P = 1$, $K_I = 0$, and $K_D = 0$ and connect the controller to the system
2. Increase the proportional gain (K_P') while forcing small disturbances until the system oscillates with a constant amplitude then record the K_P' and T_C
3. Calculate K_P, K_I, and K_D as follows :

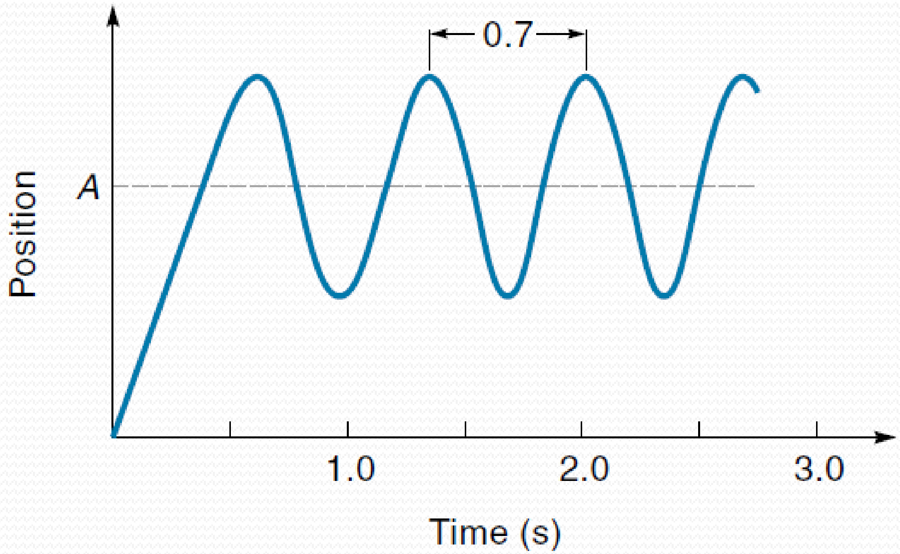
$$K_P = 0{,}6\, K_P'$$

$$K_I = \frac{2}{T_C}$$

$$K_D = \frac{T_C}{8}$$

Example

- A control system is to be tuned using the continuous-cycle method. Initial settings were $K_P = 1$, $K_I = 0$, and $K_D = 0$. By experiment it was found that the system first went into constant amplitude oscillations when $K_P = 4$. Determine a set of values for K_P, K_I, and K_D.



Example

Solution

From the response graph, we see that the period of oscillation is about 0.7 s. Therefore, T_C = 0,7 s, and we can calculate the parameters K_P, K_I, and K_D :

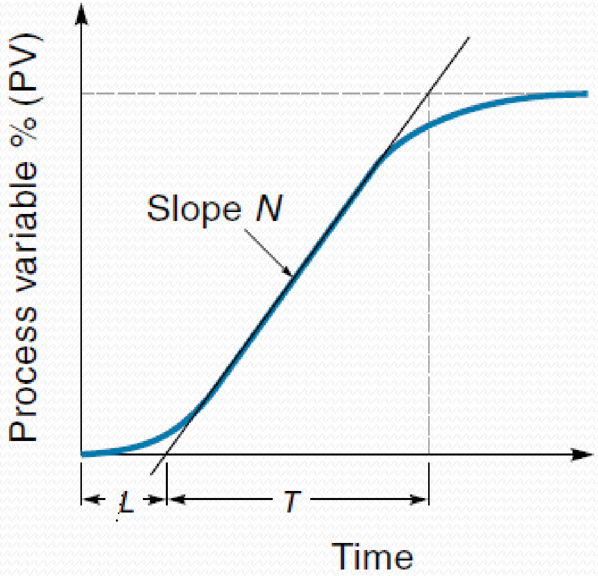
$$K_P = 0{,}6\, K_P' = 0{,}6 \times 4 = 2{,}4$$

$$K_I = \frac{2}{T_C} = \frac{2}{0{,}7\,\mathrm{s}} = \frac{2{,}9}{\mathrm{s}}$$

$$K_D = \frac{T_C}{8} = \frac{0{,}7\,\mathrm{s}}{8} = 0{,}09\,\mathrm{s}$$

Reaction-curve method

- Does not require driving the system to oscillation
- Instead, the feedback loop is opened, and the controller is manually directed to output a small step function to the actuator



Reaction-curve method

1. Draw a line tangent to the rising part of the response curve. This line defines the lag time (L) and rise time (T) values. Lag time is the time delay between the controller output and the controlled variable's response.

Reaction-curve method

2. Calculate the slope of the curve :

$$N = \frac{\Delta PV}{T}$$

where

N = slope of the system-response curve

ΔPV = change in the process variable, as reported by the sensor (in percentage)

T = rise time, from response curve

Reaction-curve method

3. Calculate the PID constants :

$$K_P = \frac{1{,}2\Delta CV}{NL} \qquad K_I = \frac{1}{2L} \qquad K_D = 0{,}5L$$

where

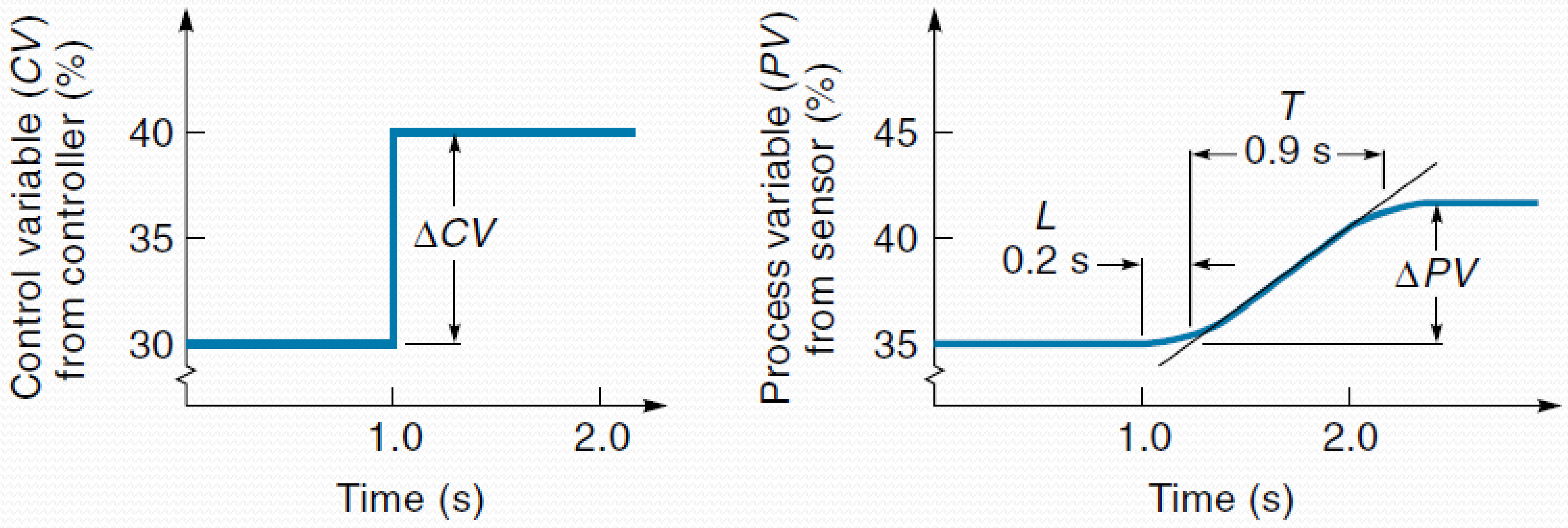
ΔCV = percent step change the in control variable (output of controller)

N = slope

L = lag time

Example

- The reaction-curve method is used to tune a PID control system. The controller was directed to output a small step-function signal. The system response was recorded. Determine values of K_P, K_I, and K_D.



(a) Step input (b) Response to step input

Example

Solution

We see that the signal from the controller (CV) was a step of 10% (40% – 30% = 10%). The PV went from 35 to 42% for a change of 7%. After draw the tangent line, we read from the graph L = 0,2 s and T = 0,9 s.

Next calculate the slope of the response :

$$N = \frac{\Delta PV}{T} = \frac{7\%}{0{,}9\ \text{s}} = 7{,}8\% / \text{s}$$

Example

Now we can calculate the PID constants :

$$K_P = \frac{1{,}2\Delta CV}{NL} = \frac{1{,}2 \times 10\%}{7{,}8\% / \mathrm{s} \times 0{,}2\,\mathrm{s}} = 7{,}7$$

$$K_I = \frac{1}{2L} = \frac{1}{2 \times 0{,}2\,\mathrm{s}} = 2{,}5 / \mathrm{s}$$

$$K_D = 0{,}5L = 0{,}5 \times 0{,}2\,\mathrm{s} = 0{,}1\,\mathrm{s}$$

Objectives Completed

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